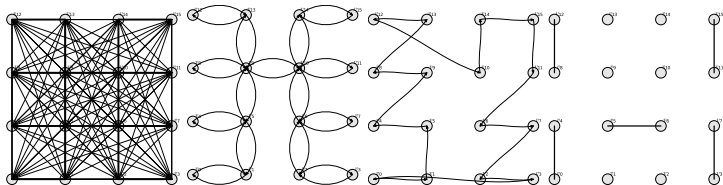


Алгоритм Кристофидеса

Н. Н. Кузюрин С. А. Фомин

8 марта 2023 г.



Определение

Эйлеровым путем в графе называется произвольный путь, проходящий через каждое ребро графа в точности один раз.

Определение

Замкнутый эйлеров путь называется **эйлеровым обходом** или **эйлеровым циклом**.

Определение

Эйлеров граф — граф, в котором существует эйлеров обход.

Теорема

Эйлеров цикл существует $\Leftrightarrow$ граф связный и все его вершины четной степени.

```
def EulerCircuitExists(G):  
    for v in G.nodes():      #Проверка необходимых  
        if G.degree(v) % 2 != 0: # условий существования  
            return False  
    return True
```

Доказательство.

Достаточность $\leftarrow$ алгоритм нахождения эйлерова пути.

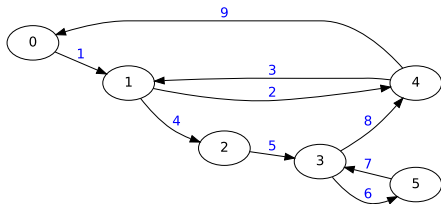
Необходимость:

если вершина v в эйлеровом обходе k раз
 $\Rightarrow$ степень этой вершины в графе $2k$.



Алгоритм нахождения Эйлера цикла

```
def EulerCircuit(G):# next(iter(G))
    EP = [ 0 ]
    while G.number_of_edges() > 0:
        for i, v in enumerate(EP):
            # куда бы добавить цикл?
            if G.degree(EP[i]) > 0:
                break
        while G.degree(v) > 0:
            #пока не в «тупике»
            w = next(G.neighbors(v))
            G.remove_edge(v, w)
            i += 1
            EP.insert(i, w)
            v = w # и повторяем все с w
    return EP
```



[0] + (0 : <12340>) -> [0, 1, 2, 3, 4, 0]

[0, 1, 2, 3, 4, 0] + (1 : <41>) -> [0, 1, 4, 1, 2, 3, 4, 0]

[0, 1, 4, 1, 2, 3, 4, 0] + (3 : <53>) -> [0, 1, 4, 1, 2, 3, 5, 3, 4, 0]

[0, 1, 4, 1, 2, 3, 5, 3, 4, 0]

Задача

«Коммивояжер», «TSP^a». Заданы неориентированный граф из n вершин-городов, и $d_{ij} \equiv d(v_i, v_j)$ — положительные целые расстояния между городами.

Чему равна наименьшая возможная длина гамильтонова цикла (кольцевого маршрута, проходящего по одному разу через все города)? т. е. нужно найти минимально возможное значение суммы

$$\min_{p \in \begin{pmatrix} 1 & 2 & \dots & n \\ . & . & . & . \end{pmatrix}} \sum_{i=1}^{n-1} d_{p_i, p_{i+1}} + d_{p_n, p_1}, \quad (1)$$

где минимум берется по всем перестановкам p чисел $1, \dots, n$.

^aВ англоязычной литературе — Traveling Salesman Problem.

Определение

*Задача коммивояжера (задача 1 «TSP») называется **метрической**, если для матрицы расстояний выполнено неравенство треугольника:*

$$\forall i, j, k \quad d_{ik} \leq d_{ij} + d_{jk}.$$

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Метрическая задача коммивояжера $\mathcal{NP}$ -полна.

Доказательство.

- 1 «Гамильтонов цикл» (единичные веса) — $\mathcal{NP}$ -полна.

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Доказательство.

- ❶ «Гамильтонов цикл» (единичные веса) — $\mathcal{NP}$ -полна.
- ❷ Дополним до полного графа ребрами с весом 2.

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- ❸ Граф с весами 1 и 2 — метрический.

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Доказательство.

- ❶ «Гамильтонов цикл» (единичные веса) — $\mathcal{NP}$ -полна.
- ❷ Дополним до полного графа ребрами с весом 2.
- ❸ Граф с весами 1 и 2 — метрический.
- ❹ Если оптимум не включает в себя ребра с весом 2 — то это гамильтонов цикл для исходного графа.



Задача

«Минимальное остовное дерево» (*Minimum Spanning Tree*)

Задан связный неориентированный граф $G = (V, E)$, где $V = \{v_1, \dots, v_n\}$ — множество вершин, $|V| = n$, E — множество ребер между ними, и весовая функция $w : E \rightarrow \mathbb{Z}^+$ ^a

Требуется найти наименьший возможный вес остовного дерева, т. е.

$$\min \sum_{(i,j) \in T} w(v_i, v_j), \quad (2)$$

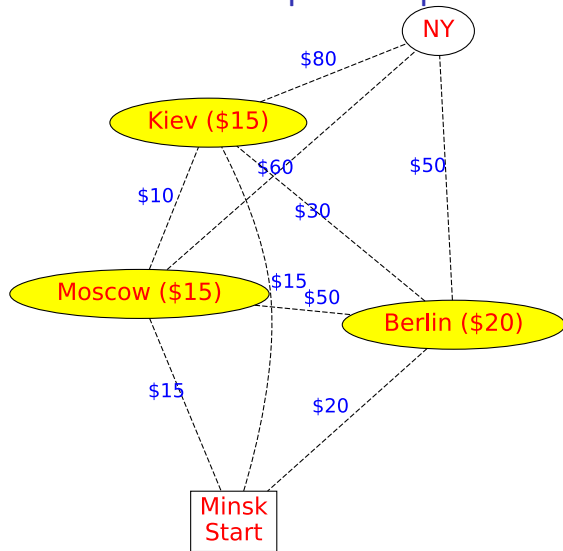
где минимум берется по всем остовным деревьям на n вершинах (по всем множествам T из $(n - 1)$ дуг, связывающим все n вершин в единую сеть).

^aМожно вводить положительные целые веса на ребрах, как $w_{ij} \equiv w(v_i, v_j)$.

Алгоритм Прима

```
def MST_Prim(G, s):  
    MST = {} #хэш (узел: предшественник)  
    ToVisit = {s: 0} # граничащие с MST, (узел: стоимость)  
    Ancestor = {s: s} #хэш: вершины из которых включают другие  
    while ToVisit: # пока есть непосещенные вершины  
        v = argmin(ToVisit) # ближайшая достижимая вершина  
        MST[v] = Ancestor[v] # запоминаем, откуда пришли  
        del ToVisit[v]  
        del Ancestor[v] # больше не посещать  
        for w in G.neighbors(v): # для всех соседей вершины v  
            if w not in MST: # которые еще не в MST  
                # обновляем стоимость включения в MST  
                if (w not in ToVisit  
                    or G[v][w]["weight"] < ToVisit[w]):  
                    ToVisit[w] = G[v][w]["weight"]  
                    Ancestor[w] = v  
    return MST
```

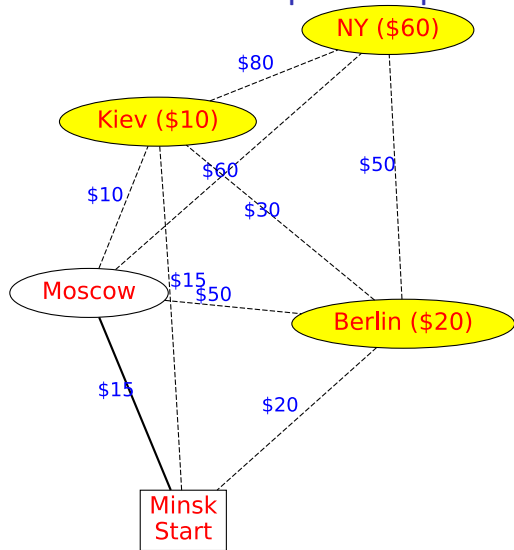
Выполнение алгоритма Прима



Итерация № 1

Стоимость MST: 0

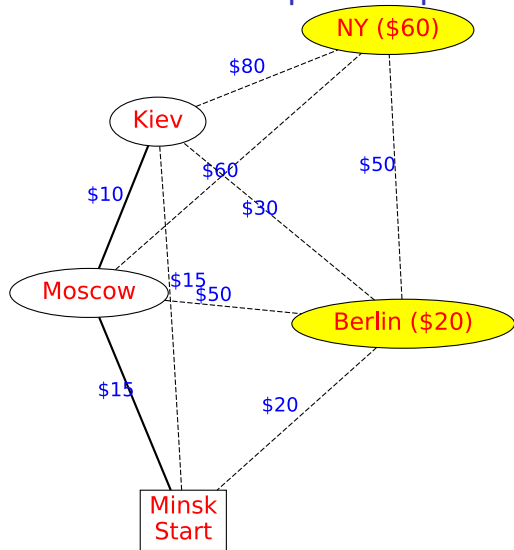
Выполнение алгоритма Прима



Итерация № 2

Стоимость MST: 15

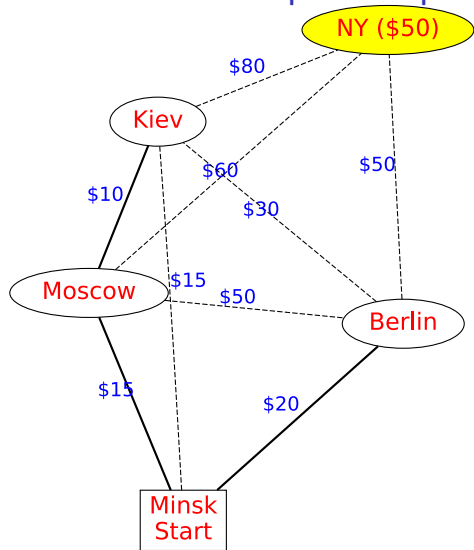
Выполнение алгоритма Прима



Итерация № 3

Стоимость MST: 25

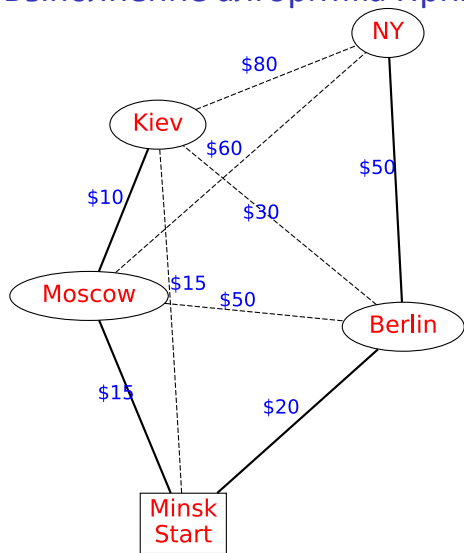
Выполнение алгоритма Прима



Итерация № 4

Стоимость MST: 45

Выполнение алгоритма Прима

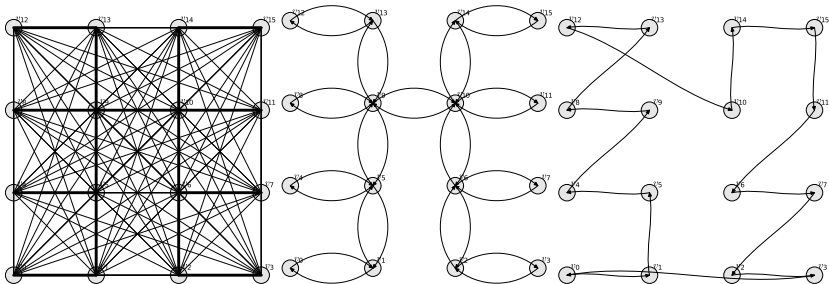


Итерация № 5

Стоимость MST: 95

Приближенный алгоритм-1 для метрической «TSP»

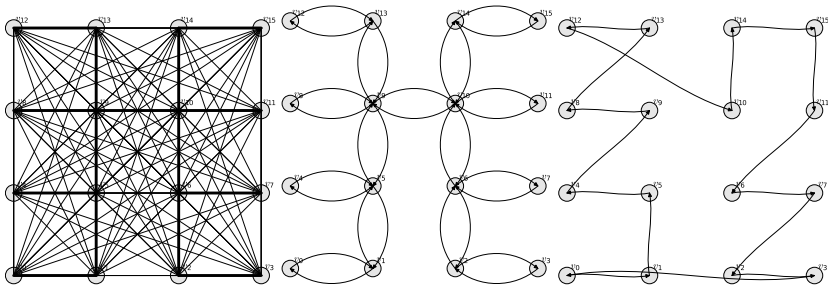
- 1 Найти минимальное остовное дерево T с матрицей весов $[d_{ij}]$.
- 2 Построить эйлеров граф G и эйлеров обход в G , продублировав все ребра дерева T .
- 3 Из эйлерова маршрута (обхода) гамильтонов цикл строится путем последовательного вычеркивания вершин, встретившихся ранее.



Алгоритм-1 2-приближенный

Доказательство.

- 1 Кратчайший гамильтонов цикл не меньше MST.
- 2 Длина эйлерова маршрута в G равна двум MST.
- 3 Неравенство треугольника $\Rightarrow$ полученный гамильтонов цикл не длиннее эйлерова обхода $\Rightarrow$ не длиннее двух MST $\Rightarrow \dots$



Определение

Паросочетание^a — подмножество ребер графа, такое, что никакие два ребра из этого подмножества не инцидентны какой-либо одной вершине.

^a*Matching* в англоязычной литературе.

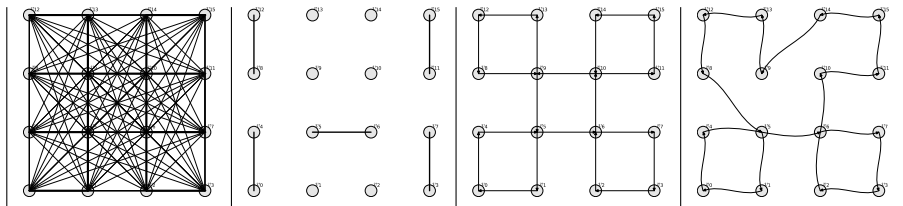
Определение

Совершенное паросочетание^a — паросочетание, покрывающее все вершины графа.

^a*Perfect matching* в англоязычной литературе.

Алгоритм Кристофидеса для метрической «TSP»

- 1 Найти MST T .
- 2 $N(T)$ — вершины нечетной степени в T
- 3 M — кратчайшее совершенное паросочетание с множеством вершин $N(T)$.
- 4 G_E эйлеров граф с вершинами $\{v_1, \dots, v_n\}$ и ребрами $T \cup M$.
- 5 Найти эйлеров обход P_E в G_E .
- 6 Построить гамильтонов цикл P_H из P_E последовательным вычеркиванием посещенных вершин.

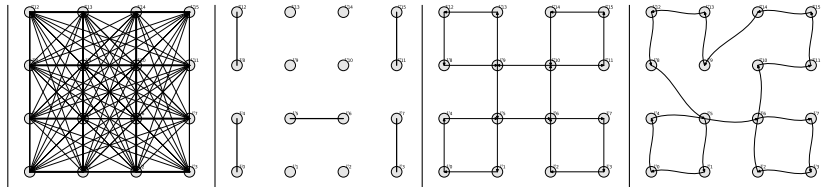


Алгоритм Кристофидеса — $\frac{3}{2}$ -приближенный

G_E — эйлеров и связен. P_H — результат. P_H^* — оптимум. c — стоимость

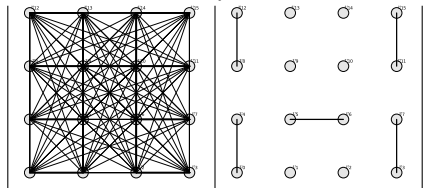
$$c(P_H) \leq c(P_E) = c(G_E) = c(T) + c(M).$$

$$c(T) \leq c(P_H^*) \leq c(P_H).$$



Алгоритм Кристофидеса — $\frac{3}{2}$ -приближенный

$i_1, i_2, \dots, i_{2m}$ — вершины нечетной степени в T (в порядке P_H^*).



Рассмотрим паросочетания:

$$M_1 = \{i_1, i_2\}, \{i_3, i_4\}, \dots, \{i_{2m-1}, i_{2m}\},$$

$$M_2 = \{i_2, i_3\}, \{i_4, i_5\}, \dots, \{i_{2m}, i_1\}.$$

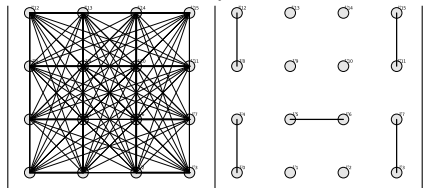
Из неравенства треугольника $\Rightarrow c(P_H^*) \geq c(M_1) + c(M_2)$.

M — кратчайшее $\Rightarrow c(M_1) \geq c(M), c(M_2) \geq c(M)$.

$$c(P_H^*) \geq 2c(M) \Rightarrow c(M) \leq \frac{1}{2}c(P_H^*).$$

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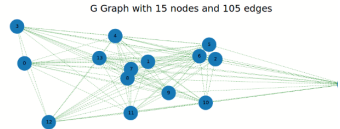
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$$c(P_H) \leq c(T) + c(M) \leq c(P_H^*) + \frac{1}{2}c(P_H^*) = \frac{3}{2}c(P_H^*).$$

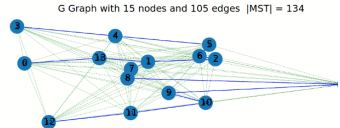
Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):  
    ... MST = nx.minimum_spanning_tree(G)  
    ... L = G.copy()  
    ... L.remove_nodes_from([v for v, degree in MST.degree  
    ... if not (degree % 2)])  
    ... min_matching = nx.min_weight_matching(L, maxcardinality=T  
    ... MG = nx.MultiGraph()  
    ... MG.add_edges_from(MST.edges)  
    ... MG.add_edges_from(min_matching)  
  
    ... euler_cycle = list(nx.eulerian_circuit(MG))  
    ... cristo_tour = []  
    ... start_node = euler_cycle[0][0]  
    ... cristo_tour.append(start_node)  
    ... for u, v in euler_cycle:  
    ...     if v not in cristo_tour:  
    ...         cristo_tour.append(v)  
    ... cristo_tour.append(start_node)  
  
    ... greedy_tsp_tour = nx_app.greedy_tsp(G)  
    ... sa_tsp_tour = nx_app.simulated_annealing_tsp(G,  
    ...     init_cycle=cristo_tour, max_iteration  
  
    ... return cristo_tour, MST
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G Graph with 15 nodes and 105 edges |MST| = 134

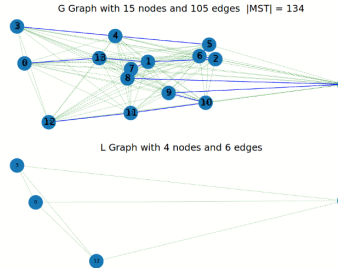


L Graph with 15 nodes and 105 edges



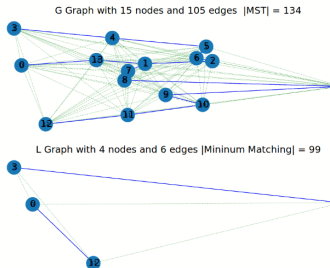
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    ... if not (degree % 2)])  
    min_matching = nx.min_weight_matching(L, maxcardinality=1)  
    MG = nx.MultiGraph()  
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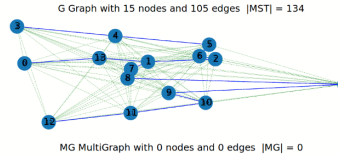
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    MG.add_edges_from(min_matching)  
  
    euler_cycle = list(nx.eulerian_circuit(MG))  
    cristo_tour = []  
    start_node = euler_cycle[0][0]  
    cristo_tour.append(start_node)  
    for u, v in euler_cycle:  
        if v not in cristo_tour:  
            cristo_tour.append(v)  
    cristo_tour.append(start_node)  
  
    greedy_tsp_tour = nx_app.greedy_tsp(G)  
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,  
        init_cycle=cristo_tour, max_iteration=10000)  
  
    return cristo_tour, MST
```



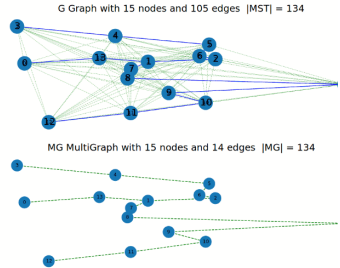
Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):  
    MST = nx.minimum_spanning_tree(G)  
    L = G.copy()  
    L.remove_nodes_from([v for v, degree in MST.degree  
                        if not (degree % 2)])  
    min_matching = nx.min_weight_matching(L, maxcardinality=True)  
    MG = nx.MultiGraph()  
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        init_cycle=cristo_tour, max_iteration=  
  
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```



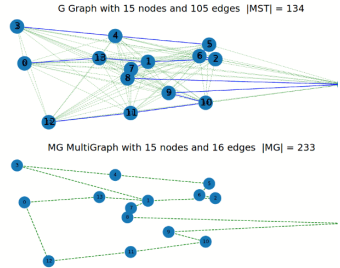
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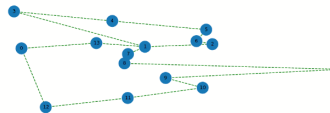
    greedy_tsp_tour = nx.app.greedy_tsp(G)
    sa_tsp_tour = nx.app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
cristo_tour															
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

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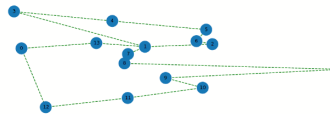
    greedy_tsp_tour = nx.app.greedy_tsp(G)
    sa_tsp_tour = nx.app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 0 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
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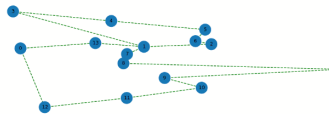
    greedy_tsp_tour = nx.app.greedy_tsp(G)
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G Graph with 15 nodes and 105 edges |TSPC| = 0 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



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euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
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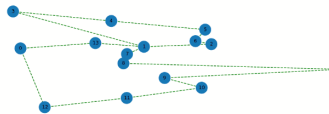
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G Graph with 15 nodes and 105 edges |TSPC| = 0 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



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cristo_tour	0														
greedy_tsp_tour															
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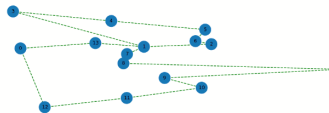
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G Graph with 15 nodes and 105 edges |TSPC| = 0 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



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cristo_tour	0														
greedy_tsp_tour															
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Выполнение алгоритма Кристофидеса

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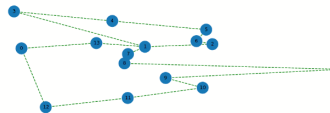
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```

G Graph with 15 nodes and 105 edges |TSPC| = 0 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
cristo_tour	0														
greedy_tsp_tour															
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Выполнение алгоритма Кристофидеса

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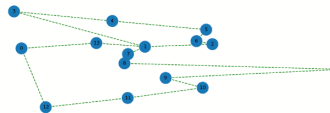
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```

G Graph with 15 nodes and 105 edges |TSPC| = 61 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
cristo_tour	0	12													
greedy_tsp_tour															
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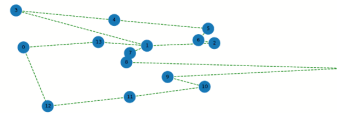
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G Graph with 15 nodes and 105 edges |TSPC| = 61 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



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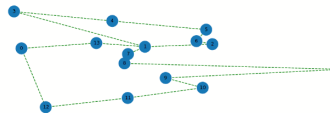
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G Graph with 15 nodes and 105 edges |TSPC| = 61 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



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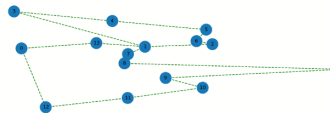
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G Graph with 15 nodes and 105 edges |TSPC| = 61 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



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cristo_tour	0	12	11												
greedy_tsp_tour															
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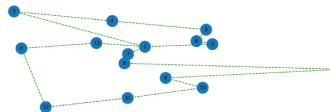
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    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 61 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
cristo_tour	0	12	11												
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                        if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

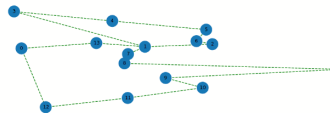
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 61 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
cristo_tour	0	12	11												
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
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    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

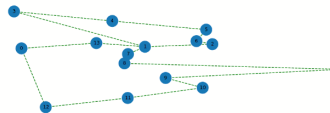
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
        init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 61 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5	6	2	1
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
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        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

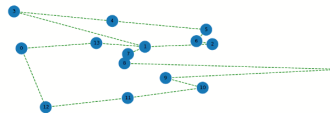
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
        init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 61 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
cristo_tour	0	12	11	10											
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
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    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

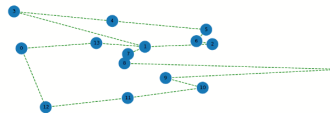
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

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```

G Graph with 15 nodes and 105 edges |TSPC| = 61 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
cristo_tour	0	12	11	10											
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
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    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

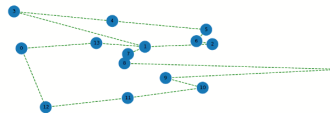
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

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```

G Graph with 15 nodes and 105 edges |TSPC| = 61 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
cristo_tour	0	12	11	10	9	14									
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
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            cristo_tour.append(v)
            cristo_tour.append(start_node)

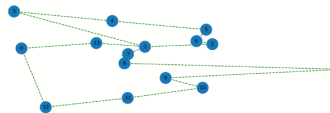
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

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```

G Graph with 15 nodes and 105 edges |TSPC| = 61 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
cristo_tour	0	12	11	10	9										
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
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        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

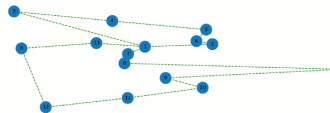
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 61 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
cristo_tour	0	12	11	10	9										
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
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    L.remove_nodes_from([v for v, degree in MST.degree
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    MG.add_edges_from(min_matching)

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    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
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        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

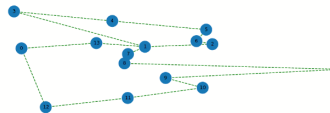
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 75 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
cristo_tour	0	12	11	10	9	14									
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
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    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
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            cristo_tour.append(v)
        cristo_tour.append(start_node)

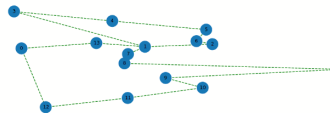
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 75 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
cristo_tour	0	12	11	10	9	14									
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

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        cristo_tour.append(start_node)

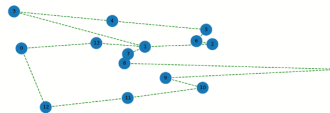
    greedy_tsp_tour = nx_app.greedy_tsp(G)
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                                                init_cycle=cristo_tour, max_iterations=1000)

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```

G Graph with 15 nodes and 105 edges |TSPC| = 75 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
cristo_tour	0	12	11	10	9	14									
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
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    MST = nx.minimum_spanning_tree(G)
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        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

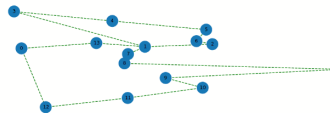
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 75 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1
cristo_tour	0	12	11	10	9	14	8								
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
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        if v not in cristo_tour:
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        cristo_tour.append(start_node)

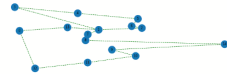
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iteration=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges (TSPC) = 75 (MST) = 134



MG MultiGraph with 15 nodes and 16 edges (MG) = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
euler_cycle	0	12	13	11	15	10	9	8	34	14	8	6	7	7	1	1
cristo_tour	0	12	13	11	15	10	9	8	34	14	8	6	7	7	1	1
greedy_tsp_tour																
sa_tsp_tour																

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                        if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

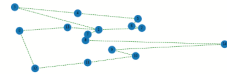
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
        init_cycle=cristo_tour, max_iteration=10000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges (TSPC) = 75 (MST) = 134



MG MultiGraph with 15 nodes and 16 edges (MG) = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
greedy_tsp_tour	0	12	13	11	15	10	9	8	34	14	8	6	7	1	1	1
sa_tsp_tour	0	12	13	11	15	10	9	8	34	14	8	6	7	1	1	1

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                          if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

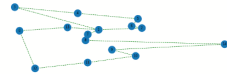
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iteration=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges (TSPC) = 75 (MST) = 134



MG MultiGraph with 15 nodes and 16 edges (MG) = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
euler_cycle	0	12	13	11	15	10	9	8	34	14	8	6	7	1	1	1
cristo_tour	0	12	13	11	15	10	9	8	34	14	8	6	7	1	1	1
greedy_tsp_tour																
sa_tsp_tour																

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                          if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

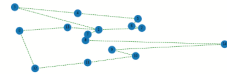
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iteration=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges (TSPC) = 75 (MST) = 134



MG MultiGraph with 15 nodes and 16 edges (MG) = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
euler_cycle	0,12	12,11	11,10	10,9	9,14	14,8	8,7	7,1	1,3	3,4	4,5	5,6	6,2	2,1	1,13	13,0
cristo_tour	0	12	11	10	9	14	8	7								
greedy_tsp_tour																
sa_tsp_tour																

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                        if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

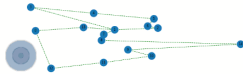
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
        init_cycle=cristo_tour, max_iteration=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges (TSPC) = 75 (MST) = 134



MG MultiGraph with 15 nodes and 16 edges (MG) = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
euler_cycle	0	12	13	11	15	10	9	8	34	14	8	6	7	1	1	1
cristo_tour	0	12	13	11	15	10	9	8	34	14	8	6	7	1	1	1
greedy_tsp_tour																
sa_tsp_tour																

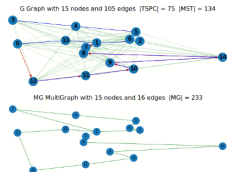
Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                          if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iteration=1000)

    return cristo_tour, MST
```



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
euler_cycle	0	12	13	11	15	10	9	8	34	14	8	6	7	1	1	1
cristo_tour	0	12	13	11	15	10	9	8	34	14	8	6	7	1	1	1
greedy_tsp_tour																
sa_tsp_tour																

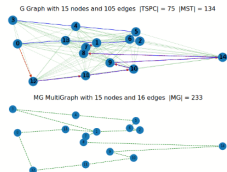
Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                          if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
            cristo_tour.append(start_node)

    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
euler_cycle	0.12	12.11	11.10	10.9	9.14	14.8	8.7	7.1	1.3	3.4	4.5	5.6	6.7	2.5	1.13	13.0
cristo_tour	0	12	11	10	9	14	8	7	1							
greedy_tsp_tour																
sa_tsp_tour																

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                          if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

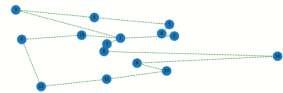
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iteration=10000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges [TSPC] = 75 [MST] = 134



MG MultiGraph with 15 nodes and 16 edges [MG] = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
euler_cycle	0.12	12.11	11.10	10.9	9.14	14.8	8.7	7.1	1.3	3.4	4.5	5.6	6.2	2.1	1.13	13.0
cristo_tour	0	12	11	10	9	14	8	7	1							
greedy_tsp_tour																
sa_tsp_tour																

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                          if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

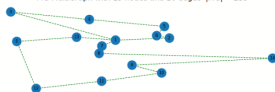
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iteration=10000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges [TSPC] = 113 [MST] = 134



MG MultiGraph with 15 nodes and 16 edges [MG] = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
euler_cycle	0,12	12,11	11,10	10,9	9,14	14,8	8,7	7,1	1,3	3,4	4,5	5,6	6,2	2,1	1,13	13,0
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5	6	2	1	13
greedy_tsp_tour																
sa_tsp_tour																

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                        if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

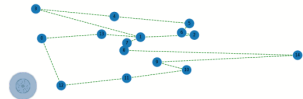
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iteratio

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges [TSPC] = 113 [MST] = 134



MG MultiGraph with 15 nodes and 16 edges [MG] = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
euler_cycle	0.12	12.11	11.10	10.9	9.14	14.8	8.7	7.1	1.3	3.4	4.5	5.6	6.2	2.1	1.13	13.0
cristo_tour	0	12	11	10	9	14	8	7	1	3						
greedy_tsp_tour																
sa_tsp_tour																

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                        if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

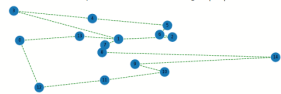
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iteration=10000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges [TSPC] = 113 [MST] = 134



MG MultiGraph with 15 nodes and 16 edges [MG] = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
euler_cycle	0.12	12.11	11.10	10.9	9.14	14.8	8.7	7.1	1.3	3.4	4.5	5.6	6.2	2.1	1.13	13.0
cristo_tour	0	12	11	10	9	14	8	7	1	3						
greedy_tsp_tour																
sa_tsp_tour																

Выполнение алгоритма Кристофидеса

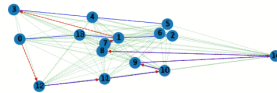
```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                        if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
    cristo_tour.append(start_node)

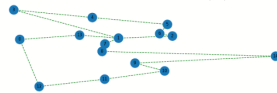
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
        init_cycle=cristo_tour, max_iteration=10000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges [TSPC] = 113 [MST] = 134



MG MultiGraph with 15 nodes and 16 edges [MG] = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
euler_cycle	0.12	12.11	11.10	10.9	9.14	14.8	8.7	7.1	1.3	3.4	4.5	5.6	6.2	2.1	1.13	13.0
cristo_tour	0	12	11	10	9	14	8	7	1	3						
greedy_tsp_tour																
sa_tsp_tour																

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                          if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

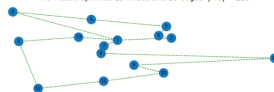
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iteration=10000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges [TSPC] = 125 [MST] = 134



MG MultiGraph with 15 nodes and 16 edges [MG] = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
euler_cycle	0.12	12.11	11.10	10.9	9.14	14.8	8.7	7.1	1.3	3.4	4.5	5.6	6.2	2.1	1.13	13.0
cristo_tour	0	12	11	10	9	14	8	7	1	3	4					
greedy_tsp_tour																
sa_tsp_tour																

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                          if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

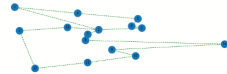
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iteration=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges (TSPC) = 125 (MST) = 134



MG MultiGraph with 15 nodes and 16 edges (MG) = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
euler_cycle	0	12	13	11	15	10	16	9	8	3	14	8	7	1	13	15
cristo_tour	0	12	13	11	15	10	16	9	8	3	14	8	7	1	13	15
greedy_tsp_tour																
sa_tsp_tour																

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                          if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

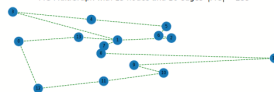
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iteration=10000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges [TSPC] = 125 [MST] = 134



MG MultiGraph with 15 nodes and 16 edges [MG] = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
euler_cycle	0.12	12.11	11.10	10.9	9.14	14.8	8.7	7.1	1.3	3.4	4.5	5.6	6.2	2.1	1.13	13.0
cristo_tour	0	12	11	10	9	14	8	7	1	3	4					
greedy_tsp_tour																
sa_tsp_tour																

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                        if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
            cristo_tour.append(start_node)

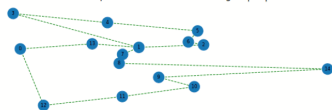
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
        init_cycle=cristo_tour, max_iterations=10000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 125 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1, 13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4				
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
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        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

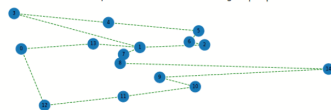
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
        init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 125 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0,12	12,11	11,10	10,9	9,14	14,8	8,7	7,1	1,3	3,4	4,5	5,6	6,2	2,1	1,13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4				
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
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            cristo_tour.append(v)
        cristo_tour.append(start_node)

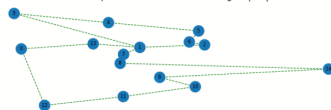
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 125 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1, 13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4				
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                        if not (degree % 2)])
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    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
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        if v not in cristo_tour:
            cristo_tour.append(v)
            cristo_tour.append(start_node)

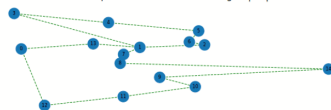
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
        init_cycle=cristo_tour, max_iterations=10000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 135 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1, 13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5			
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                        if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

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    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

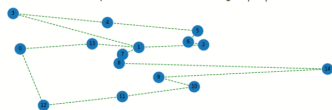
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
        init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 135 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1, 13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5			
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
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    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

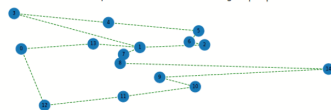
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 147 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1, 13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5	6	2	1
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
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    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

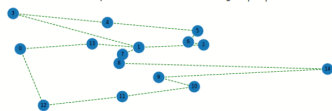
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 147 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1, 13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5	6	2	1
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
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    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

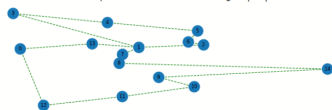
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
        init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 147 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1, 13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5	6		
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
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        cristo_tour.append(start_node)

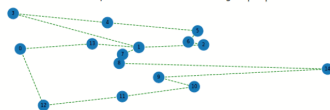
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
        init_cycle=cristo_tour, max_iteration=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 147 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1, 13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5	6		
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
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    MST = nx.minimum_spanning_tree(G)
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            cristo_tour.append(v)
        cristo_tour.append(start_node)

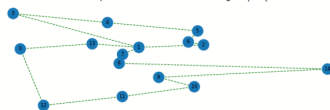
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                                                init_cycle=cristo_tour, max_iterations=1000)

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```

G Graph with 15 nodes and 105 edges |TSPC| = 147 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1, 13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5	6	2	1
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

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def tsp_cristofides(G):
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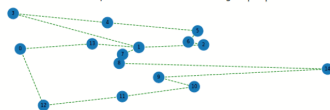
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        init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 147 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1, 13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5	6	2	
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
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        if v not in cristo_tour:
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        cristo_tour.append(start_node)

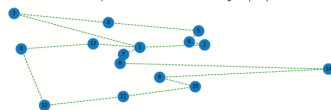
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
        init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 147 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1, 13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5	6	2	
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
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    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

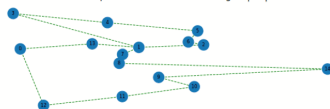
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
        init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 147 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1, 13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5	6	2	
greedy_tsp_tour															
sa_tsp_tour															

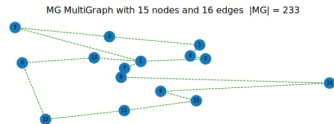
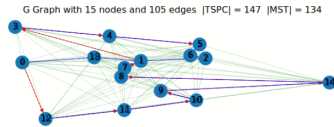
Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
    L.remove_nodes_from([v for v, degree in MST.degree
                          if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
    cristo_tour.append(start_node)

    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                  init_cycle=cristo_tour, max_iterations=10000)

    return cristo_tour, MST
```

[illegible]

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
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    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

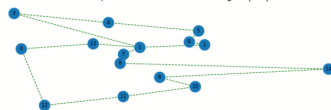
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 155 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0,12	12,11	11,10	10,9	9,14	14,8	8,7	7,1	1,3	3,4	4,5	5,6	6,2	2,1	1,13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5	6	2	13
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
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    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
            cristo_tour.append(start_node)

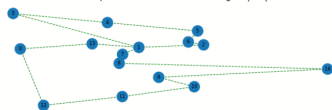
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 155 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1, 13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5	6	2	13
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
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                        if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
        cristo_tour.append(start_node)

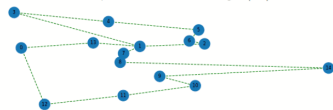
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
        init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 155 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0, 12	12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 2	2, 1	1, 13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5	6	2	13
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
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    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
    cristo_tour.append(start_node)

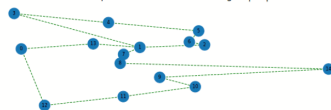
    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```

G Graph with 15 nodes and 105 edges |TSPC| = 155 |MST| = 134



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0,12	12,11	11,10	10,9	9,14	14,8	8,7	7,1	1,3	3,4	4,5	5,6	6,2	2,1	1,13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5	6	2	13
greedy_tsp_tour															
sa_tsp_tour															

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
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                        if not (degree % 2)])
    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

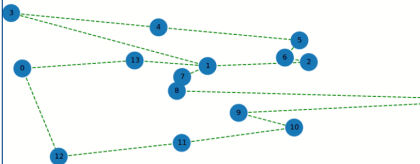
    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
    cristo_tour.append(start_node)

    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
        init_cycle=cristo_tour, max_iteration=10000)

    return cristo_tour, MST
```



MG MultiGraph with 15 nodes and 16 edges $|MG| =$



1	2	3	4	5	6	7	8	9	10	11	12
12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 12
12	11	10	9	14	8	7	1	3	4	5	6

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
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                        if not (degree % 2)])
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    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

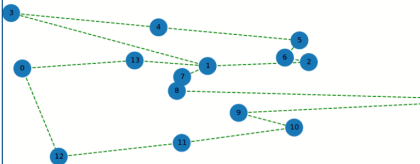
    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
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    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iteration=1000)

    return cristo_tour, MST
```



MG MultiGraph with 15 nodes and 16 edges $|MG| =$



1	2	3	4	5	6	7	8	9	10	11	12
12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6
12	11	10	9	14	8	7	1	3	4	5	

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
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    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
    MG.add_edges_from(min_matching)

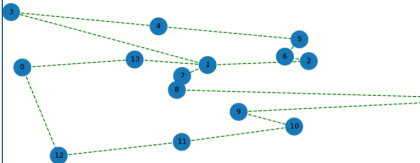
    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
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    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iteration=1000)

    return cristo_tour, MST
```



MG MultiGraph with 15 nodes and 16 edges $|MG| =$



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 7	7, 8	8, 9	9, 10
12	11	10	9	14	8	7	1	3	4	5	6	7	8	9

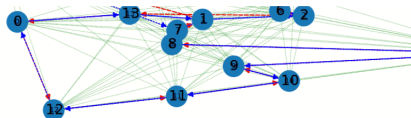
Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
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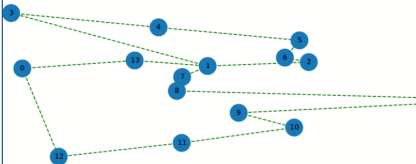
    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
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    cristo_tour.append(start_node)

    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iteration=1000)

    return cristo_tour, MST
```



MG MultiGraph with 15 nodes and 16 edges $|MG| =$



1	2	3	4	5	6	7	8	9	10	11	12
12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6
12	11	10	9	14	8	7	1	3	4	5	

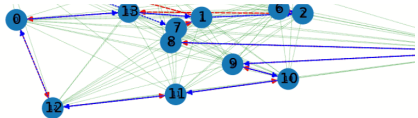
Выполнение алгоритма Кристофидеса

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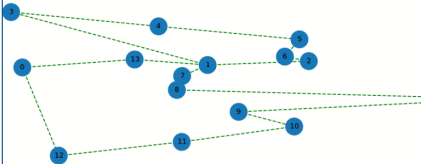
    euler_cycle = list(nx.eulerian_circuit(MG))
    cristo_tour = []
    start_node = euler_cycle[0][0]
    cristo_tour.append(start_node)
    for u, v in euler_cycle:
        if v not in cristo_tour:
            cristo_tour.append(v)
    cristo_tour.append(start_node)

    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                  init_cycle=cristo_tour, max_iterations=1000)

    return cristo_tour, MST
```



MG MultiGraph with 15 nodes and 16 edges $|MG| =$



1	2	3	4	5	6	7	8	9	10	11	12
12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 12
12	11	10	9	14	8	7	1	3	4	5	
12	11	10	9	14	8	7	1	3	4	5	

Выполнение алгоритма Кристофидеса

```
def tsp_cristofides(G):
    MST = nx.minimum_spanning_tree(G)
    L = G.copy()
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    min_matching = nx.min_weight_matching(L, maxcardinality=True)
    MG = nx.MultiGraph()
    MG.add_edges_from(MST.edges)
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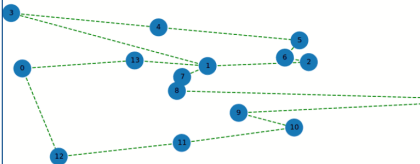
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    start_node = euler_cycle[0][0]
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    greedy_tsp_tour = nx_app.greedy_tsp(G)
    sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
                                                init_cycle=cristo_tour, max_iteration=1000)

    return cristo_tour, MST
```



MG MultiGraph with 15 nodes and 16 edges $|MG| =$



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
12, 11	11, 10	10, 9	9, 14	14, 8	8, 7	7, 1	1, 3	3, 4	4, 5	5, 6	6, 7	7, 8	8, 9	9, 10
12	11	10	9	14	8	7	1	3	4	5	6	7	8	9

Выполнение алгоритма Кристофидеса

```

... cristo_tour.append(v)
... cristo_tour.append(start_node)

... greedy_tsp_tour = nx_app.greedy_tsp(G)
... sa_tsp_tour = nx_app.simulated_annealing_tsp(G,
... init_cycle=cristo_tour, max_iteration=
... return cristo_tour, MST

```

```

if __name__ == "__main__":

```

```

... G = subway_graph()
... cristo_tour, mst = tsp_cristofides(G)

```

```

... cristo_edges = list(nx.utils.pairwise(cristo_tour))
... for u, v, d in G.edges(data=True):
...     if not ((u, v) in cristo_edges or (u, v) in cristo_edges)
...         d['weight'] = 2

```

```

... cristo_tour = tsp_cristofides(G)
... pass

```

```

...

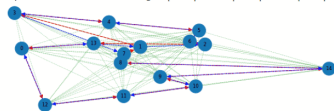
```

```

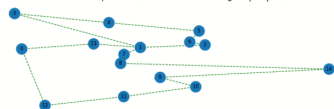
ToDo:

```

G Graph with 15 nodes and 105 edges |TSPC| = 155 |TSPG| = 118 |TSPS| =



MG MultiGraph with 15 nodes and 16 edges |MG| = 233



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
euler_cycle	0.12	12.11	11.10	10.9	9.14	14.8	8.7	7.1	1.3	3.4	4.5	5.6	6.2	2.1	1.13
cristo_tour	0	12	11	10	9	14	8	7	1	3	4	5	6	2	13
greedy_tsp_tour	0	13	1	2	14	9	4	3	7	8	14	9	10	11	12
sa_tsp_tour	0	12	11	10	9	14	8	7	1	3	4	5	6	2	13

<http://discopal.ispras.ru/>

Вопросы?